Measurement of the Dielectric Properties of Biological Substances Using an Improved Open-Ended Coaxial Line Resonator Method

DEMING XU, LIPING LIU, AND ZHIYAN JIANG

Abstract —An improved open-ended coaxial line resonator method for measuring the complex permittivity of biological substances is presented. By considering the end radiation losses and higher order mode effects, the upper frequency limit is extended from 4 GHz to 11 GHz. A novel $\lambda/4$ open-ended coaxial line resonator, which uses a thin copper diaphragm for a coupling structure, was developed. Experiments show that the structure is very flexible, convenient, and reliable. Experimental results for several canine organs as well as human skin are given. The method has useful applications to microwave medicine and bioelectromagnetic studies.

I. INTRODUCTION

STUDIES OF the microwave and radio-frequency properties of biological substances date back to the 1950's or earlier. Knowledge of their properties, as well as the frequency and temperature dependence, is of great importance in both basic and applied research, such as investigations of the interaction mechanisms of electromagnetic fields with biological systems, absorption of biological tissues in electromagnetic fields, maximum permissible levels (MPL's) of microwave and radio-frequency radiation, and heating distribution in microwave diagnostics. In the last 20 years, increasing interest in this field has led to a need for more accurate data on the permittivity of biological materials, especially *in vivo* data. It is apparent that the development of appropriate measurement techniques suitable for wide frequency band coverage has become an important task in bioelectromagnetics.

Coaxial line sensors have come into wide use in measuring the dielectric properties of biomaterials. However, until recently both reflection and resonator methods were restricted to the frequency range below 4 GHz [1], [2] because of the influences of radiation losses and higher order modes at the open end of the coaxial line.

We have analyzed these influences and expressed these concisely in another paper [3]. The influence of radiation losses can be expressed as a conductance, which has been derived as a polynomial; the effect of higher order modes is attributed to the variation of fringe capacitance at the

open end of the coaxial line. The results show that the capacitance is a function of frequency. In this paper, a more extensive description of the resonator method is presented. Selected biological tissues and phantom muscle are measured in the frequency range 0.1–11 GHz. The results are in good agreement with the published data.

II. PRINCIPLE OF MEASUREMENT

A. Equivalent Circuit

Fig. 1 shows an open-ended coaxial line resonator. The sample to be tested is placed in contact with the open end of the line. Fig. 2 shows the equivalent circuit; $C(\bar\epsilon), C_f$ represent the field concentrations in region 2 and region 1, respectively. $G(\bar\epsilon)$ is a conductance representing the radiation losses in the dielectric sample. For a lossy medium, both $C(\bar\epsilon)$ and $G(\bar\epsilon)$ are complex values. ΔC is the coupling capacitance of the air gap at the inner conductor and L_r, C_r, and G_r are the equivalent resonator parameters in the unloaded condition.

When region 2 in Fig. 1 is free space, the parameter $G(\epsilon_0)$ and $B = \omega C(\epsilon_0)$ can be calculated using the following formulas given by Marcuvitz [4]:

$$G/Y_0 = \frac{1}{\ln a/b} \int_0^{\pi/2} \frac{1}{\sin\theta} \left[J_0(ka\sin\theta) \right.$$
$$\left. - J_0(kb\sin\theta) \right]^2 d\theta \quad (1)$$

$$B/Y_0 = \frac{1}{\ln a/b} \int_0^{\pi} \left[2\operatorname{Si}\left(k\sqrt{a^2 + b^2 - 2ab\cos\varphi} \right) \right.$$
$$\left. - \operatorname{Si}\left(2ka\sin\frac{\varphi}{2} \right) - \operatorname{Si}\left(2kb\sin\frac{\varphi}{2} \right) \right] d\varphi \quad (2)$$

where $J_0(x)$ is the Bessel function, $\operatorname{Si}(x)$ is the sinusoidal integral, and k denotes $2\pi/\lambda$.

Expanding $J_0(\sin\theta)$ into a Maclaurin series and integrating gives

$$G/Y_0 = \frac{1}{\ln a/b}$$
$$\cdot \left[G_1(f^4) + G_2(f^6) + G_3(f^8) + G_4(f^{10}) + \cdots \right]$$
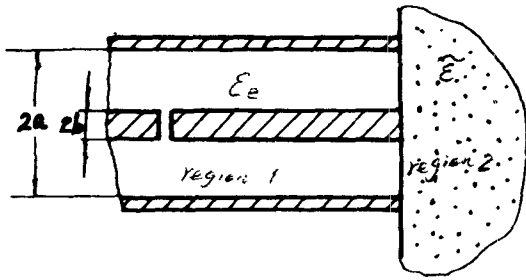$$\quad (3)$$

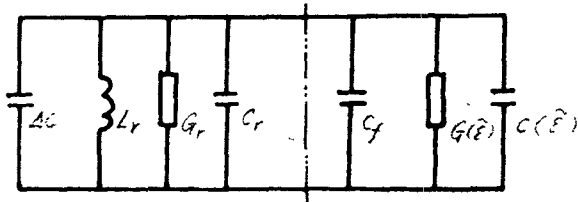Fig. 1   A coaxial line resonator terminated with sample.

Fig. 2.   Equivalent circuit of the resonator.

where

$$G_1(f^4) = \frac{2}{3}(a^2 - b^2)^2 \frac{\pi^4}{\lambda^4}$$

$$G_2(f^6) = -\frac{1}{15}(a^2 - b^2)(a^4 - b^4)\frac{\pi^6}{\lambda^6}$$

$$G_3(f^8) = \frac{16}{35}\left[\frac{1}{16}(a^4 - b^4)^2 + \frac{1}{18}(a^2 - b^2)(a^6 - b^6)\right]\frac{\pi^8}{\lambda^8}$$

$$\tag{4}$$

$$\cdots\cdots\cdots\cdots\cdots$$

and

$$f = v_0/\lambda.$$

Considering the influences of higher order modes and using a variational method, the fringe capacitance can be expressed as follows:

$$C_0(f) = C_{00} + \Delta C_0 f^2 + \Delta C_1 f^4 + \cdots \tag{5}$$

where C_{00} represents the static fringe capacitance and $\Delta C_0, \Delta C_1, \cdots$ are coefficients, which depend on the line dimensions.

If region 2 is filled with a dielectric material, the open-ended coaxial line can be considered as an antenna. According to the Deschampes theorem, the input admittance of the antenna is given by

$$Y(\omega, \tilde{\epsilon}\epsilon_0) = \sqrt{\tilde{\epsilon}}\, Y(\sqrt{\tilde{\epsilon}}\,\omega, \epsilon_0) = G(\tilde{\epsilon}) + j\omega C(\tilde{\epsilon}) \tag{6}$$

where

$$G(\tilde{\epsilon}) = \tilde{\epsilon}^{5/2} G_1(f^4) + \tilde{\epsilon}^{7/2} G_2(f^6) + \tilde{\epsilon}^{9/2} G_3(f^8)$$
$$+ \tilde{\epsilon}^{11/2} G_4(f^{10}) + \cdots \tag{7}$$

$$C(\tilde{\epsilon}) = \tilde{\epsilon} C_{00} + \Delta C_0 \tilde{\epsilon}^2 f^2 + \Delta C_1 \tilde{\epsilon}^4 f^4 + \cdots$$
$$\doteq \tilde{\epsilon} C_{00} + \Delta C_0 \tilde{\epsilon}^2 f^2. \tag{8}$$

Equations (7) and (8) give the expressions for radiation conductance G and fringe capacitance C as a function of

frequency and the complex permittivity of material to be tested. It has been shown that good accuracy can be obtained using only two terms in the computation of $C(\tilde{\epsilon})$ [Eq. (8)]. To illustrate the accuracy of (7), we cite the following data. For the case of $\epsilon_r = 80$, $f = 12.4$ GHz, and a 6.4-mm-diameter line, using the first seven terms in (7) gives an error in $G(\tilde{\epsilon})$ of less than 10 percent. For a 3.2-mm line, the error is less than 4.5 percent using the first three terms. In another case, if $\epsilon_r = 3$, $f = 12.4$ GHz, the error for a 6.4-mm line using the first three terms is less than 5 percent, and for a 3.2-mm line using the first term is less than 2 percent.

B. Principle of Measurement

If region 2 is free space, the radiation losses and higher order mode effects can be neglected. The resonant condition of the resonator is then given by

$$Y_{01} c \tan(\beta_0 l_{\text{eff}}) = \omega_0 (C_0 + C_f). \tag{9}$$

Solving for l_{eff}, the effective length of the resonator, gives

$$l_{\text{eff}} = \frac{1}{\beta_0}\left\{ n\pi + \tan^{-1}\left[\frac{Y_{01}}{\omega_0(C_0 + C_f)}\right]\right\} \tag{10}$$

where ω_0 is the resonant frequency, $\beta_0 = 2\pi f_0\sqrt{\epsilon_e}/v_0$, and n is the mode number TEM$_n$, which can be determined by measuring two adjacent resonant frequencies of the resonator. The quality factor is expressed as

$$Q_0 = \frac{\omega_0 C_{e0}}{G_{e0}} \tag{11}$$

where C_{e0} and G_{e0} are the total capacitance and conductance of the resonant circuit when it resonates at ω_0.

If region 2 is filled with a dielectric material with $\tilde{\epsilon} = \epsilon_r(1 - j\tan\delta)$, assume that the resonant frequency and the quality factor are ω_1 and Q_1, respectively. The radiation conductance and the fringe capacitance expressed by (7) and (8) can be rewritten as

$$G(\tilde{\epsilon}) = G_1(f^4)\tilde{\epsilon}^{5/2} + G_2(f^6)\tilde{\epsilon}^{7/2} + G_3(f^8)\tilde{\epsilon}^{9/2} + \cdots$$
$$= g - jp \tag{12}$$

$$B(\tilde{\epsilon}) = \omega_1 C(\tilde{\epsilon}) = \omega_1\left(C_{00}\tilde{\epsilon} + \Delta C_0 f_1^2 \tilde{\epsilon}^2\right)$$

$$= \omega_1\left[C_{00}\epsilon_r + \Delta C_0 f_1^2(1 - \tan^2\delta)\right]$$

$$- j\omega_1\left(C_{00}\epsilon_r \tan\delta + 2\Delta C_0 f_1^2 \epsilon_r^2 \tan\delta\right). \tag{13}$$

Similarly, the resonant condition is given by

$$Y_{01} c \tan(\beta_1 l_{\text{eff}})$$

$$= \omega_1\left[C_f + \Delta C_0 f_1^2 \epsilon_r^2(1 - \tan^2\delta) + C_{00}\epsilon_r\right] - p \tag{14}$$

so

$$\epsilon_r = \frac{Y_{01} c \tan\beta_1 l_{\text{eff}} + p}{\omega_1 C_{00}} - \frac{C_f + \Delta C_0 f_1^2 \epsilon_r^2(1 - \tan^2\delta)}{C_{00}}. \tag{15}$$

The loss tangent of the material can be found from the

following equations:

$$\tan\delta = k\left(\frac{1}{Q_1} - \frac{1}{Q_a} - \frac{1}{Q_r}\right) \qquad (16)$$

where the quality factor Q_1 is given by

$$Q_1 = \omega_1 C_{el}/G_{el} \qquad (17)$$

and

$$k = \frac{C_{el}}{\left(\epsilon_r C_{00} + 2\,\Delta C_0 f_1^2 \epsilon_r^2\right)} \qquad (18)$$

$$Q_a = \frac{Q_0 \omega_1 C_{el}}{\omega_0 C_{e0}} \qquad (19)$$

$$Q_r = \frac{\omega_1 C_{el}}{g} \qquad (20)$$

where C_{el} and G_{el} are total capacitance and conductance of the resonant circuit when the sensor is surrounded with dielectric material and resonates at the frequency ω_1. Therefore, ϵ_r and $\tan\delta$ can be determined from the measurable quantities using (15) and (16).

III. MEASUREMENT TECHNIQUE

A. Resonator and Coupling Structure

A $\lambda/2$ coaxial line resonator has been described by Tanabe and Joines [2]. By breaking the center conductor of the coaxial line, an air gap was formed and the energy is coupled through this gap (see Fig. 1). A coupling capacitance ΔC was introduced in the equivalent circuit (see Fig. 2). In this paper, we designed a $\lambda/4$ coaxial line resonator. A 3.2-mm line is used and the coupling structure is formed by a thin copper diaphragm, which is sandwiched between the two joint interfaces of an APC-7 connector (Fig. 3). The connector connects the sensor to the measurement system. By changing the shape and the size of the diaphragm, we can achieve different coupling factors. The presence of the diaphragm gives rise to magnetic energy storage in the vicinity of the diaphragm. It is equivalent to a shunt inductive reactance ΔL in the equivalent circuit. The effect of ΔC or ΔL is to elongate or shorten the length of the resonator. We have introduced the term "effective length" in (10) to account for this effect. Results of three reference samples using three different diaphragm shapes are given in Table I. It shows that the shape of the diaphragm has almost no influence on the measurement results of ϵ_r and $\tan\delta$. Experiment shows that this coupling structure is very flexible, convenient, and reliable.

B. Measurement Equipment

The equipment used in the measurement is shown in Fig. 4. A network analyzer is used to indicate resonance and half power frequency points. Frequency is measured using a frequency counter.

Because the resonator acts in the strong coupling state, the unloaded Q factor is strongly dependent on the coupling factor k'. The coupling factor can be determined by
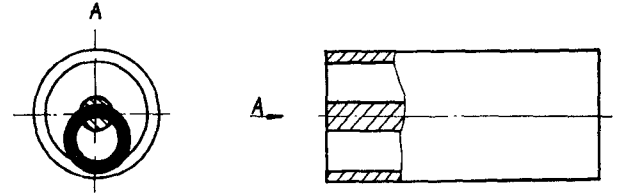

Fig. 3 Figure of the coupling structure.

TABLE I
COMPARISON OF MEASURED RESULTS OBTAINED BY USING THREE DIFFERENT COUPLING STRUCTURES

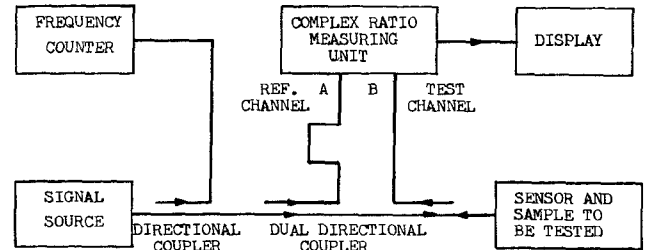| coupling structure	material	frequency (GHz)	ϵ_r	$\tan\delta$
	Distilled water	9.01746	63.41	0.475
	Skin(hand dorsum)	9.06939	21.53	0.528
	Methanol	9.19990	7.36	1.216
	Distilled water	9.05505	64.25	0.504
	Skin(hand dorsum)	9.10980	23.72	0.657
	Methanol	9.24234	7.39	1.061
	Distilled water	9.03283	62.01	0.522
	Skin(hand dorsum)	9.09247	23.01	0.510
	Methanol	9.22023	8.33	1.022
Standard deviation	Distilled water		1.13?	0.023
	Skin(hand dorsum)		0.352	0.065
	Methanol		0.321	0.084

Fig. 4. Experimental arrangement.

measuring the standing wave ratio ρ_0 at the resonant frequency:

$$k' = \begin{cases} 1 + \rho_0 & \text{(strong coupling)} \\ 1 + 1/\rho_0 & \text{(weak coupling)}. \end{cases} \qquad (21)$$

In order to measure the higher loss materials, the resonator should be nearly critically coupled.

C. Measurement of C_0, C_f, and ΔC_0

C_0 and C_f can be measured using either of two methods [1]. In the first method, the capacitances are determined from the input reflection coefficient when dielectrics with well-known properties are in contact with the sensor. In the second method, a coaxial line resonator is formed and

TABLE II
FRINGE CAPACITANCES OF COAXIAL LINES WITH DIAMETERS OF
6.4 mm AND 3.2 mm

Diameter	C_{T1}(pF)	C_{T2}(pF)	C_0(pF)	C_f(pF)	C_g(pF)
6.4mm	0.0382	0.0718	0.0330	0.0052	0.0005
3.2mm	0.0246	0.0462	0.0211	0.0036	0.0001

by measuring the resonant frequency, the capacitances can be determined.

We used the second method in this paper. When the resonator end is open- and short-circuited, respectively, the total capacitance in (9), $C_T = C_0 + C_f$, can be determined from the measured resonant frequencies. If the measurements are performed for the open line as well as terminated by a known dielectric, C_0 and C_f can be separated from the measured capacitance C_T. In order to determine the ΔC_0, we measured the total capacitances C_T at two different frequencies f_1, f_2 and from (8) obtain

$$\Delta C_0 = \frac{C_T(f_1) - C_T(f_2)}{f_1^2 - f_2^2}. \qquad (22)$$

The two frequencies f_1 and f_2 must be separated as far as possible in order to increase the accuracy of ΔC_0. By using mercury we can achieve a good short circuit and thereby obtain a sufficiently accurate measure of the total capacitance C_T.

Table II shows the measured capacitances of two coaxial lines with diameters of 6.4 mm and 3.2 mm at 8.99 GHz. Teflon was used as the reference sample.

IV. MEASUREMENT RESULTS

Seven organs of a dog, i.e., muscle, skin, liver, kidney, heart, and gray matter and white matter of the brain, were measured *in vitro*. The samples were surgically prepared and immersed in saline solution. All measurements were finished within 12 hours. The room temperature was controlled within $20 \pm 1°C$. The sample surface was larger than 10×10 mm^2 and the thickness was over 5 mm.

Results of measurements in the UHF to *X*-band frequency range are given in Table III. It shows that the measured results agree well with the reference data [5], [6]. Fig. 5 shows the results for muscle as a function of frequency (see also Table III). The results for phantom muscle are also given (Table IV).

Table V gives a comparison of measured results with published data for some dielectric materials. It can be seen that for high-loss dielectric materials, the measured values of loss tangent are close to the published data. Hence the method is suitable for biological materials. However, for low-loss materials, the differences are large. This is because of inadequate resolution of the quality factor. We measured the dielectric properties of human skin (hand dorsum) and compared these results with the data given by Schwan and by Tanabe (Fig. 6). It shows that in the frequency range below 4 GHz, the results obtained from Tanabe's

TABLE III
MEASUREMENT RESULTS OF DOG TISSUES

Frequency		muscle	heart	kidney	liver	skin	white matter	grey matter
0.1GHz	ε'	72.74	61.25	68.30	56.83	38.30	59.86	77.53
	ε''	126.04	82.23	96.17	57.04	51.89	62.64	111.78
0.5GHz	ε'	61.47	56.40	59.49	57.53	45.43	37.34	63.55
	ε''	36.26	28.47	32.77	22.89	19.67	14.09	30.67
0.75GHz	ε'	58.34	58.12	57.52	56.28	40.71	43.50	60.25
	ε''	30.22	26.03	24.54	19.77	12.46	14.61	24.27
1.0GHz	ε'	51.97	53.00	55.77	54.62	41.05	41.19	58.79
	ε''	20.30	21.23	17.15	15.38	11.56	14.27	17.61
2.0GHz	ε'	53.92	52.49	53.84	51.94	39.51	42.83	49.65
	ε''	17.03	17.01	16.71	15.03	11.21	11.49	18.36
3.0GHz	ε'	52.92	51.98	53.01	49.12	39.19	38.17	51.96
	ε''	15.02	15.11	15.02	14.09	10.96	8.53	13.59
4.0GHz	ε'	49.24	51.78	48.67	45.49	34.13	36.09	61.66
	ε''	13.97	13.05	12.83	13.53	8.68	8.05	11.76
6.0GHz	ε'	44.10	41.46	43.00	42.14	33.07	31.32	44.52
	ε''	22.42	23.02	21.05	19.25	14.21	11.96	26.40
9.0GHz	ε'	40.82	39.57	43.39	38.32	28.08	30.08	44.76
	ε''	26.48	26.18	29.19	25.04	16.92	13.39	29.51
11.0GHz	ε'	35.23	28.75	30.71	32.13	25.19	23.19	30.01
	ε''	31.89	21.17	25.08	24.51	16.87	13.37	24.73

TABLE IV
MEASUREMENT RESULTS OF PHANTOM MUSCLE

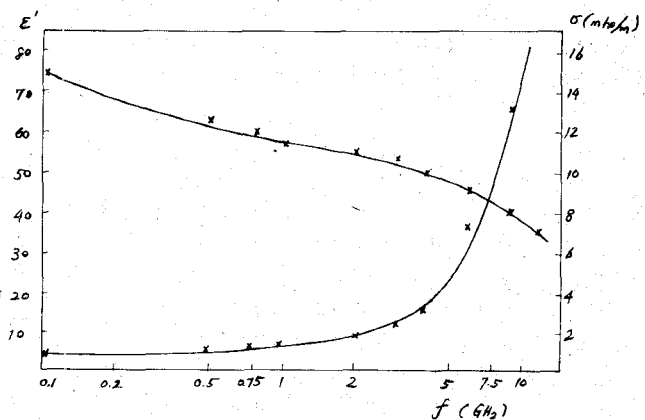|	ε'	ε''
Measured Data	46.34	15.53
Reference Data	47.00	15.93

Fig. 5. Dielectric properties of dog muscle as a function of frequency.

method and the method presented in this paper agree well with the curve given by Schwan. In the frequency range 4–11 GHz, there are significant errors in the conductivity data of Tanabe and Joines due to their neglect of radiation losses and higher order mode effects. After improving on Tanabe's method, we have obtained much better agree-

TABLE V
COMPARISON OF MEASURED DATA FROM SEVERAL MATERIALS WITH PUBLISHED DATA

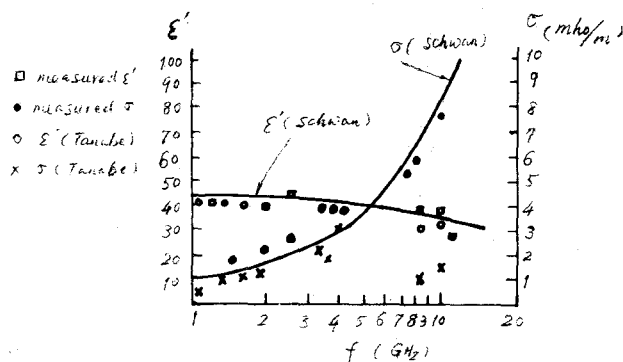| Dielectric Material	Frequency (GHz)	Measured Data ε_r	Measured Data $\tan\delta$	Published Data ε_r	Published Data $\tan\delta$	Difference $\Delta\varepsilon_r/\varepsilon_r$	Difference $\Delta\tan\delta/\tan\delta$
Alumina	8.7001	8.97	$1.1\ 10^{-3}$	9.01	$2.1\ 10^{-4}$	0.004	
Quartz crystal	8.7543	4.54	$3.0\ 10^{-3}$	4.53	$1\ 10^{-4}$	0.002	
Ethanol (20°C)	9.7970	4.60	0.59	4.59	0.57	0.002	0.02
Methanol(25°C)	9.2391	7.84	1.10	8.01	1.013	0.021	0.087
Propanol(25°C)	9.2701	3.40	0.30	3.46	0.309	0.018	0.029
Distilled water (20°C)	9.3283	62.01	0.52	64.8	0.492	0.036	0.058
Skin (hand dorsum)	11.0261	30.93	0.64	±30	±0.7		

Fig. 6. Dielectric properties of human skin.

ment with Schwan's data. Tanabe and Joines estimated that with the achievable resolution of resonant frequency and Q factor, ε_r can be determined with an accuracy within 5 percent and $\tan\delta$ within 25 percent in the frequency range below 4 GHz. In this paper, the computation formula is more accurate than that used by Tanabe and Joines [2]. So, even though the measurement frequency range is extended to X-band, relative errors of $\Delta\varepsilon_r/\varepsilon_r \leqslant 5$ percent and $\Delta\tan\delta/\tan\delta \leqslant 25$ percent (for medium and high-loss materials) should still be obtainable.

V. CONCLUSION

The improved open-ended coaxial line resonator method has many advantages. By considering the influences of radiation losses and higher order modes, the measurement frequency range was successfully extended to X-band. A thin copper diaphragm coupling structure was developed. Its performance is excellent and reliable. Using this technique, data were presented on the dielectric properties of seven organs of a dog in the frequency range 0.1–11 GHz. Such data should find use in microwave medicine, bioelectromagnetics, and other fields.

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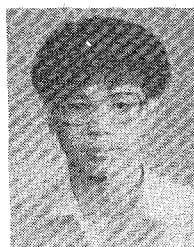
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